Model-independent approach for incorporating interference effects in collider searches for new resonances[1]

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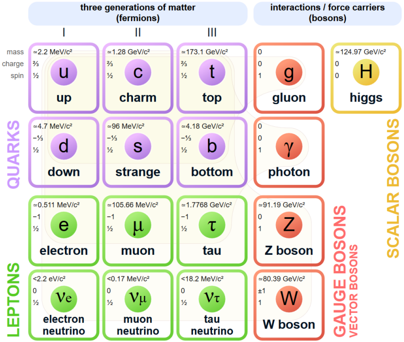
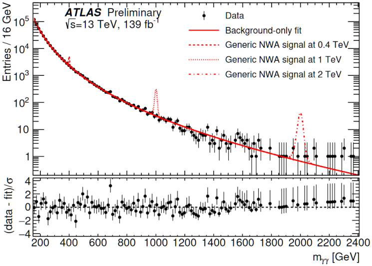
[1] Eur. Phys. J. C 80, 1174 (2020)





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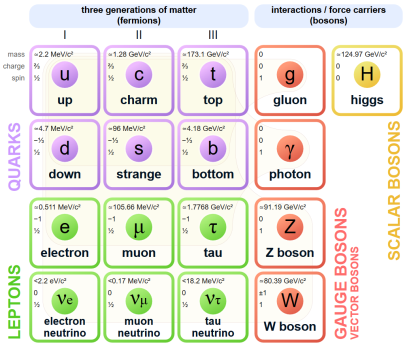

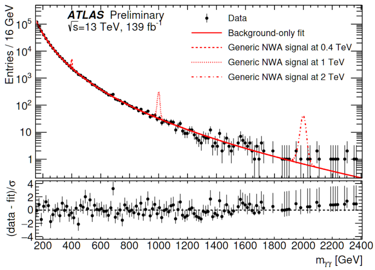

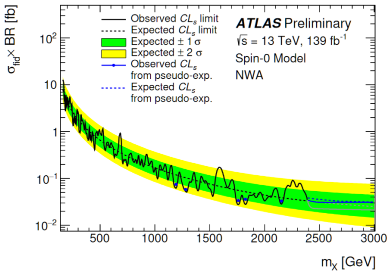

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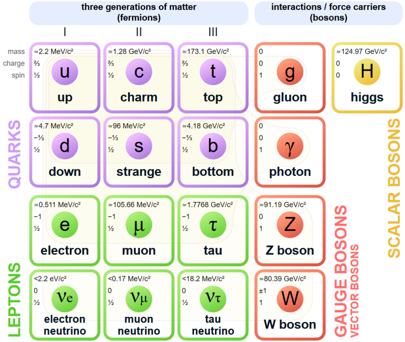

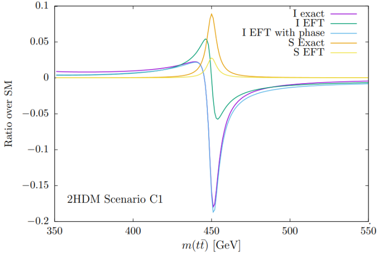

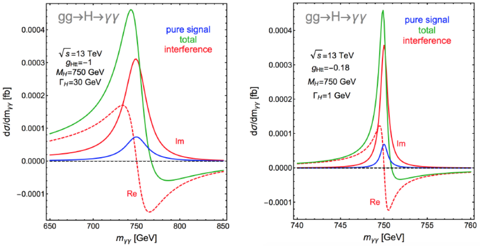

J. High Energ. Phys. 96 (2017)

J. High Energ. Phys. 07 (2016)

Introduction

We want to characterise search results in a model-independent manner, whilst accounting for possible interference contributions...

- adopt a bottom-up approach, rather than top-down
- possible BSM lineshapes are largely constrained by QFT
- functional form that provides basis for possible (S+I) lineshapes

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- single partonic channel

Partonic amplitude:

$$\bar{A}_h(q^2) = \frac{S_h(q^2)}{q^2 - m^2 + im\Gamma} + \frac{B_h(q^2)}{m^2} \tag{1}$$

- q^2 = resonance virtuality = squared invariant mass
- m = resonance mass
- Γ = resonance width
- subscript h denotes helicity configuration

Write complex phases explicitly:

$$S_h(q^2) = \left|S_h(q^2)\right| \exp\left[i\,\xi_h(q^2)\right]\ , \quad B_h(q^2) = \left|B_h(q^2)\right| \exp\left[i\,\chi_h(q^2)\right] \tag{2}$$

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Then, the square of the amplitude in eq. (1) is:

$$\left|\bar{A}_h(q^2)\right|^2 = \frac{\left|S_h(q^2)\right|^2}{(q^2-m^2)^2+m^2\Gamma^2} + \frac{\left|B_h(q^2)\right|^2}{m^4} \tag{3}$$
$$+ \frac{2}{m^2}\,\frac{\left|S_h(q^2)\right|\left|B_h(q^2)\right|}{(q^2-m^2)^2+m^2\Gamma^2}\left[(q^2-m^2)\cos\phi_h(q^2) + m\Gamma\sin\phi_h(q^2)\right]$$

where

$$\phi_h(q^2) = \xi_h(q^2) - \chi_h(q^2) \tag{4}$$

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$\rightarrow$ Eq. (3), multiplied by flux and phase space factors, is the "full" differential cross section with S+B+I contributions

Flux and phase space factors cancel out in taking the ratio:

$$\frac{\left|\bar{A}(q^2)\right|^2}{\left|\bar{B}(q^2)\right|^2} = \frac{m^4 E(q^2)}{(q^2 - m^2)^2 + m^2\Gamma^2} + \frac{m^2(q^2 - m^2)\, O(q^2)}{(q^2 - m^2)^2 + m^2\Gamma^2} + 1 \qquad (5)$$

where the "even" and "odd" functions are:

$$E(q^2) = R(q^2)^2 + 2\,\frac{\Gamma}{m}\,R(q^2)\,\sin(\phi(q^2)) \qquad (6)$$

$$O(q^2) = 2\,R(q^2)\cos(\phi(q^2)) \qquad (7)$$

with:

$$R(q^2) = \frac{|S(q^2)|}{|B(q^2)|} \qquad (8)$$

Taylor expand $E(q^2)$ and $O(q^2)$ around $q^2 = m^2$:

$$\frac{\left|\bar{A}(q^2)\right|^2}{\left|\bar{B}(q^2)\right|^2} = \frac{m^4}{(q^2 - m^2)^2 + m^2\Gamma^2} \sum_{k=0}^{\infty} \frac{a_k}{k!} \left(\frac{q^2}{m^2} - 1\right)^k + 1 \qquad (9)$$

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$$a_k = E^{(k)}(m^2) + k\, O^{(k-1)}(m^2)\,. \qquad (10)$$

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- in the neighbourhood of m^2, we can obtain an approximate solution
- truncate at some order $K \implies$ functional form, with parameters:

$$\left\{ m\,, \Gamma\,, a_0\,, \ldots a_K \right\} \qquad (11)$$

for which values can be found via a fit to data.

a_i can be related to quantities with more direct physical interpretation e.g. for $K = 2$:

$$a_0 = R^{(0)^2} + 2\,\frac{\Gamma}{m}\,R^{(0)} s_\phi^{(0)} \tag{12}$$

$$a_1 = 2\left[R^{(0)} R^{(1)} + R^{(0)} c_\phi^{(0)} + \frac{\Gamma}{m}\left(R^{(0)} s_\phi^{(1)} + R^{(1)} s_\phi^{(0)}\right)\right] \tag{13}$$

$$a_2 = 2\Bigg[R^{(1)^2} + R^{(0)} R^{(2)} + 2R^{(0)} c_\phi^{(1)} + 2R^{(1)} c_\phi^{(0)} + \frac{\Gamma}{m}\left(R^{(0)} s_\phi^{(2)} + 2R^{(1)} s_\phi^{(1)} + R^{(2)} s_\phi^{(0)}\right)\Bigg] \tag{14}$$

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- physics-driven assumptions can reduce the number of parameters e.g. assume phase to vary more slowly than $R(q^2)$
  $\rightarrow$ retain only 0^{th} order phase terms

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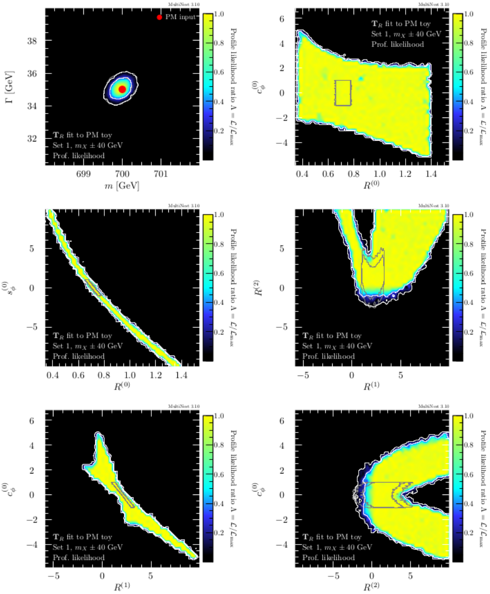
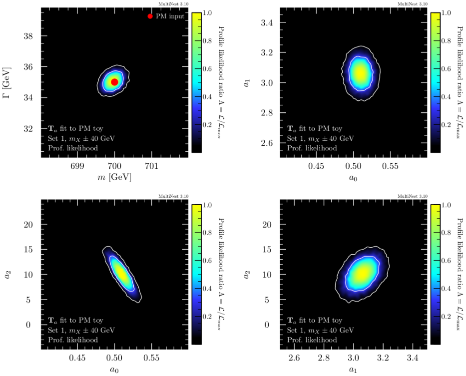
- more direct physical interpretation
- separable S and I
- underconstrained

- neater results
- computationally cheaper
- less direct interpretation


PM input
$\mathbf{T}_R$ fit to PM toy
Set 1, $m_X \pm 40$ GeV
Prof. likelihood
m [GeV]
Γ [GeV]
$R^{(0)}$
$c_\phi^{(0)}$
$s_\phi^{(0)}$
$R^{(1)}$
$R^{(2)}$
Profile likelihood ratio $\Lambda = \mathcal{L}/\mathcal{L}_{\max}$
MultiNest 3.10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Summary

- re-interpretability of resonance search results is important
- using general QFT arguments, arrived at template functional form that can describe S+I+B lineshapes
- two possible parametrisations: $\mathbf{T}_R$ and $\mathbf{T}_a$, each with pros and cons
- theorists can compute these parameters for their pet theory and compare them to experimental bounds
- invite you to check out paper for more: EPJC 80, 1174 (2020)

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Thanks!

Backup

What happened to helicity indices? / Why dintinguish c_ϕ and s_ϕ?

In the case of multiple helicity configurations, the relevant amplitudes are:

$$\left|\bar{A}(q^2)\right|^2 = \sum_{h=1}^{N} \left|\bar{A}_h(q^2)\right|^2 , \quad \left|\bar{B}(q^2)\right|^2 = \sum_{h=1}^{N} \left|\bar{B}_h(q^2)\right|^2 \tag{R.17}$$

These amount to a redefinition of $R(q^2)$ and phase-related terms:

$$R(q^2) = \sqrt{\frac{\sum_{h=1}^{N} |S_h(q^2)|^2}{\sum_{h=1}^{N} |B_h(q^2)|^2}} \tag{R.18}$$

$$c_\phi(q^2) = \frac{\sum_{h=1}^{N} \left|S_h(q^2)\right| \left|B_h(q^2)\right| \cos\phi_h(q^2)}{\sqrt{\sum_{h=1}^{N} |S_h(q^2)|^2 \sum_{h=1}^{N} |B_h(q^2)|^2}} \tag{R.19}$$

$$s_\phi(q^2) = \frac{\sum_{h=1}^{N} \left|S_h(q^2)\right| \left|B_h(q^2)\right| \sin\phi_h(q^2)}{\sqrt{\sum_{h=1}^{N} |S_h(q^2)|^2 \sum_{h=1}^{N} |B_h(q^2)|^2}} \tag{R.20}$$

$\rightarrow$ functional forms remain unchanged!