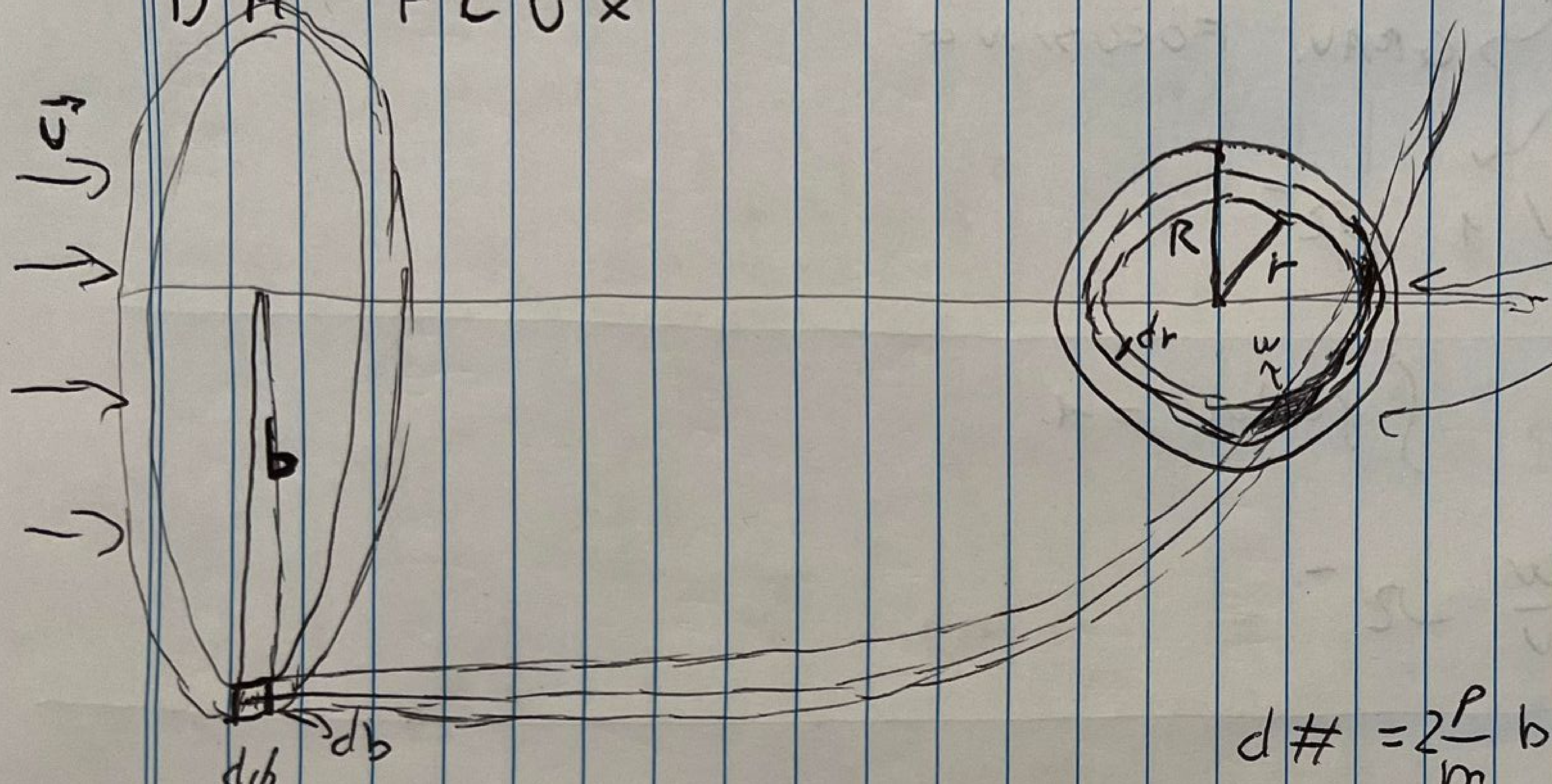
CAPTURE OF DM IN STARS AND DD RATES

DM FLUX



$$\frac{d\#}{dt} = \frac{\rho_{\otimes}}{m_{\otimes}}\, b\, db\, d\phi\, u$$

$$\frac{L}{m} = b \cdot u = w \cdot r \cdot \sin\theta$$

$$\dot{r} = \frac{dr}{dt} = w \cos\theta$$

$$d\# = 2\frac{\rho}{m}\, b\, db\, d\phi\, u \cdot \frac{dr}{\dot{r}} \qquad \sin\theta = y$$

$$= 4\pi \frac{\rho}{m}\, b\, db\, u\, \frac{dr}{\dot{r}} \qquad b\,db = \frac{w^2 r^2}{u^2}\, y\, dy$$

$$\frac{1}{2} m u^2 = \frac{1}{2} m w^2(r) \underbrace{- \frac{GMm}{r}}$$

$$\frac{1}{2} m u^2 = \frac{1}{2} m w^2 - \frac{1}{2} m v_e^2$$

$$w^2 = u^2 + v_e^2$$

$$\sin\theta = \frac{bu}{wr} \qquad \cos\theta = \sqrt{1 - \frac{b^2 u^2}{w^2 r^2}} = \sqrt{1-y^2}$$

$$d\# = 4\pi \frac{\rho}{m}\, u\, \frac{w^2 r^2}{u^2} \frac{y\, dy}{\sqrt{1-y^2}} \frac{dr}{w}$$

$$d\# = \boxed{\frac{\rho}{m} \cdot 4\pi r^2 dr\, \frac{w}{u}} \int_0^1 dy\, \frac{y}{\sqrt{1-y^2}} \rightarrow 1$$

$$R = \frac{d\#}{dt} = \frac{\rho}{m} \cdot \underbrace{4\pi r^2 dr}_{dV} \frac{w}{u} \cdot \Omega^- \rightarrow \text{INT. RATE}$$

GRAV. FOCUSING

DD $\quad v_e \sim 0 \quad \Rightarrow \frac{\rho}{m} \, dV \cdot 1 \cdot \Omega^-$

SPEED DISTR. $\quad f(u) \qquad \int f(u)\,du = 1$

$$R = \frac{\rho}{m} 4\pi r^2 dr \int du\, f(u) \frac{w}{u} \Omega^-$$

INT. RATE

$$\Omega^- = n_T \cdot w \cdot \sigma^T_{eff} \qquad \sigma^T_{eff} = \int_D d\cos\theta \frac{d\sigma^T}{d\cos\theta}$$

D : DOMAIN THAT ALLOWS CAPTURE

KIN. VAR

$\vec{s}$: speed of c.o.m.

$\vec{t}$: x speed in c.o.m. F.

$\vec{w}$: x speed lab. f.

$\vec{v}_N$: N speed lab. f.

$$\mu = \frac{m_x}{m_N} \qquad \mu_\pm = \frac{\mu \pm 1}{2}$$

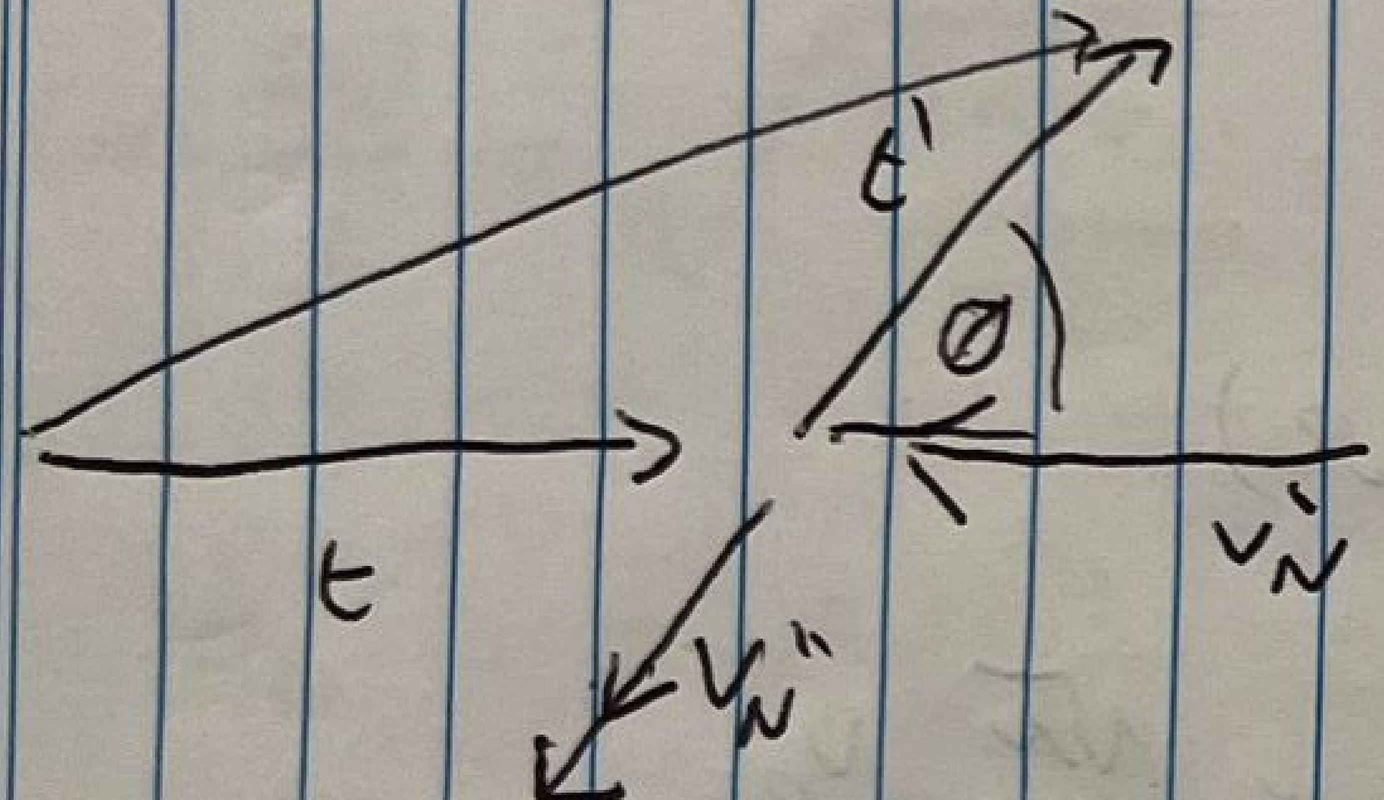
$\vec{s} = \dfrac{\mu \vec{w} + \vec{v}_N}{1+\mu}$ $\vec{t} = \dfrac{\vec{w} - \vec{v}_N}{1+\mu}$ $\vec{w} = \vec{s} + \vec{t}$

$s = \dfrac{\mu}{1+\mu} w$

$\cos\theta_{st} = \dfrac{w^2 - s^2 - t^2}{2st}$ $T \to 0 \Rightarrow v_N \to 0 \Rightarrow$ $t = \dfrac{1}{1+\mu} w$

$\cos\theta_{st} \to 1$

$\theta_{st} = 0$



$\theta = \theta_{st'}$ $\vec{v} = \vec{s} + \vec{t}\,'$

$\cos\theta_{st'} = \dfrac{v^2 - s^2 - t^2}{2st} = \dfrac{v^2(1+\mu)^2 - w^2(1+\mu^2)}{2\mu w^2}$

$\| \cos\theta$

$\Omega^- = n_T w \int \underbrace{d\cos\theta}_{\text{CHANGE}} \dfrac{d\sigma}{d\cos\theta}$ $d\cos\theta = \dfrac{\cancel{2} v\,dv\,(1+\mu)^2}{\cancel{2}\mu w^2} = \dfrac{4\mu_+^2}{\mu} \dfrac{v\,dv}{w^2}$

$= n_T \cancel{w} \dfrac{4\mu_+^2}{\mu w^{\cancel{2}}} \int v\,dv \dfrac{d\sigma^T}{d\cos\theta}$

$D = ?$

$v_{Max} = v_e$ $\cos\theta = \dfrac{v_e^2(1+\mu)^2 - (v_e^2+v^2)(1+\mu^2)}{2\mu w^2} < 1$

$v_e^2 \cancel{(1+\mu)}^2 - (v^2+v_e^2)\cancel{(1+\mu)}^2 < 0$ V

$$\cos\theta_{st'} > -1 \Rightarrow \frac{v^2(1+\mu)^2 - w^2(1+\mu^2)}{2\mu w^2} > -1$$

$$v^2(1+\mu)^2 - w^2(\mu-1)^2 > 0 \Rightarrow v^2 > \frac{\mu_-^2}{\mu_+^2} w^2 \Rightarrow v > \frac{|\mu_-|}{\mu_+} w$$

$$\Omega^- = \int_{\frac{|\mu_-|}{\mu_+} w}^{v_e} v\,dv\, n_T \frac{4\mu_+^2}{\mu w} \frac{d\sigma}{d\cos\theta}$$

$\Rightarrow$ REQUIRES $v_e > \frac{|\mu_-|}{\mu_+} w \Rightarrow v_e^2 > \frac{\mu_-^2}{\mu_+^2}(v_0^2 + u^2)$

$\Rightarrow v_e^2\left(1 - \frac{\mu_-^2}{\mu_+^2}\right) > u^2 \Rightarrow v_e^2 \frac{\mu}{\mu_+^2} > u^2 \Rightarrow u < \frac{\sqrt{\mu}}{\mu_+} v_e$

SAMPLE LOW u

$$C = \frac{\rho}{m}\int_0^R 4\pi r^2 dr \underbrace{\int_0^{\frac{\sqrt{\mu}}{\mu_+} v_e} du\, f(u) \frac{\cancel{w}}{u}}_{\int du \frac{f(u)}{u}} \cdot n_T \frac{4\mu_+^2}{\mu \cancel{w}} \int_{\frac{|\mu_-|}{\mu_+} w}^{v_e} v\,dv \frac{d\sigma}{d\cos\theta}$$

DIRECT DET.

DIFFERENT VARIABLE $\theta \to E_R$

$$E_R = \frac{q_{tr}^2}{2m_T} \qquad q_{tr} = 2t\, m \sin\frac{\theta_{sc}}{2} \qquad q_{tr}^2 = 4m^2t^2\left(1-\cos^2\frac{\theta_{sc}}{2}\right)$$

$$\cos\frac{\theta_{sc}}{2} = \sqrt{\frac{1+\cos\theta_{sc}}{2}} \qquad \Rightarrow q_{tr}^2 = 4m^2t^2\left(1-\frac{1+\cos\theta}{2}\right)$$

$$= 2m^2t^2(1-\cos\theta)$$

$T \to 0$ $$q_{tr}^2 = 2m^2\frac{w^2}{(1+\mu)^2}(1-\cos\theta) = \frac{2\underbrace{m_x^2 m_T^2}_{\to \mu_{Tx}^2} w^2}{(m_x+m_N)^2}(1-\cos\theta) = 2\mu_{Tx}^2 w^2(1-\cos\theta)$$

$$E_R = \frac{q_{tr}^2}{2m_T} = \frac{m_x^2 m_T w^2}{(m_T+m_x)^2}(1-\cos\theta) \qquad dE_R = -\frac{m_x^2 m_T w^2}{(m_T+m_x)^2}\, d\cos\theta$$

$$E_R^{MAX} = \frac{2m_x^2 m_T w^2}{(m_T+m_x)^2}$$

$$\Omega^- = m_T \cancel{w} \int_{E^R_{MIN}}^{E^R_{MAX}} \frac{(m_T+m_x)^2}{m_x^2 m_T w^{\cancel{2}}}\, dE_R \frac{d\sigma^T}{d\cos\theta}$$

$$\frac{dR}{dE_R} = \frac{\rho}{m_\chi}\int dv\, f(v)\frac{\cancel{v}}{v}\underbrace{\int dV\, n_T}_{N_T}\frac{(m_\chi+m_N)^2}{m_\chi^2 m_T \cancel{v}}\frac{d\sigma}{d\cos\theta} \qquad M_T = m_T N_T$$

$$\frac{d\bar{R}}{dE_R}(E_R) = \frac{\rho}{m_\chi}\int dv\,\frac{f(v)}{v}\frac{(m_\chi+m_N)^2}{m_\chi^2 m_T^2}\frac{d\sigma}{d\cos\theta}$$

$$E_R < E_R^{MAX} = \frac{2m_\chi^2 m_T\, w^2}{(m_T+m_\chi)^2} \xrightarrow{} v^2 = \frac{2m_\chi^2 m_T\, v^2}{(m_T+m_\chi)^2}$$

$$\Rightarrow v^2 > \frac{(m_T+m_\chi)^2}{2m_\chi^2 m_T} E_R = \frac{E_R\cdot m_T}{2\mu_{T\chi}^2} = v_{MIN} = v_{min}(E_R)$$

$$\Rightarrow \int_{v_{MIN}}^{v_{max}} dv\,\frac{f(v)}{v} \Rightarrow \text{SAMPLE LARGE } v$$

$$\frac{d\sigma^T}{d\cos\theta} = \frac{1}{32\pi}\frac{|\mathcal{M}_T|^2}{(m_\chi+m_T)^2} \qquad |\mathcal{M}_T|^2 = \frac{m_T^2}{m_N^2}\cdot C_i^N C_j^{N'} F_{ij}^{NN'}$$

COEFF. NREFT OP. FORM FACTORS