Relativistic mean-field corrections for dark matter interactions

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WIMP-nucleus scattering

The differential recoil rate in direct detection is given by

$$\frac{dR_{\chi T}}{dE_R} = \sum_T N_T \frac{\rho_\chi}{m_\chi} \int_{v_{\min}} d^3 v f(\vec{v}, t) v \frac{d\sigma_{\chi T}}{dE_R}, \qquad (1)$$

where

$$\frac{d\sigma_{\chi T}}{dE_R} = \frac{2m_T}{4\pi v^2} \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} \sum_{spins} |\mathcal{M}|^2_T \right], \qquad (2)$$

NR effective operators – spin 0 and 1/2

N. Anand *et al.*, Phys. Rev. C **89**, 065501 (2014)

In non-relativistic limit, the wavefunction of free nucleon,

$$U(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\xi \end{pmatrix} \rightarrow \begin{pmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m}\xi \end{pmatrix}, \qquad (3)$$

- $\mathcal{L}_{SI} = c_1 \bar{\chi}\chi \bar{N} N \rightarrow c_1 \xi_\chi^\dagger \mathbf{1}_\chi \xi_\chi \; \xi_N^\dagger \mathbf{1}_N \xi_N$
  $\Rightarrow \mathcal{O}_1 = \mathbf{1}_\chi \mathbf{1}_N$
- $\mathcal{L}_{SD} = c_4 \bar{\chi}\gamma_\mu\gamma_5\chi \bar{N}\gamma^\mu\gamma_5 N \rightarrow -4c_4 \xi_\chi^\dagger (S_\chi)_i \xi_\chi \; \xi_N^\dagger (S_N)_i \xi_N$
  $\Rightarrow \mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$

j	$\mathcal{L}_{\rm int}^{j}$	NR Reduction in medium $(\xi^\dagger \mathcal{O}_{eff} \xi)$	$\sum_i c_i \mathcal{O}_i$
1	$\bar{\chi}\chi\bar{N}N$	$1_\chi 1_N$ $\rightarrow$ Standard SI	$\mathcal{O}_1$
2	$i\bar{\chi}\chi\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$
3	$i\bar{\chi}\gamma^5\chi\bar{N}N$	$-i\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi$	$\mathcal{O}_{11}$
4	$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N$	$-(\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi)(\frac{\vec{q}}{m_N}\cdot\vec{S}_N)$	$\mathcal{O}_6$
5	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$
6	$\bar{\chi}\gamma^\mu\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec{q}^2}{2m_N m_M}1_\chi 1_N + 2(\frac{\vec{q}}{m_\chi}\times\vec{S}_\chi + i\vec{v}^\perp)\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$	$\{\mathcal{O}_1,\mathcal{O}_3,\mathcal{O}_4,\mathcal{O}_6\}$
7	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N$	$-2\vec{S}_N\cdot\vec{v}^\perp + 2i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_\chi})$	$\{\mathcal{O}_7,\mathcal{O}_9\}$
8	$\bar{\chi}i\gamma^\mu\chi\bar{N}\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$\mathcal{O}_{10}$
9	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}\gamma_\mu N$	$-\frac{\vec{q}^2}{2m_\chi m_M}1_\chi 1_N - 2(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot(\frac{\vec{q}}{m_N}\times\vec{S}_N + i\vec{v}^\perp)$	$\{\mathcal{O}_1,\mathcal{O}_4,\mathcal{O}_5,\mathcal{O}_6\}$
10	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$	$\{\mathcal{O}_4,\mathcal{O}_6\}$

11	$\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}\gamma_\mu\gamma_5 N$	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_N$	$\mathcal{O}_9$
12	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec{q}^2}{m_\chi m_M}-4\vec{v}^\perp\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)](\frac{\vec{q}}{m_M}\cdot\vec{S}_N)$	$\{\mathcal{O}_{10},\mathcal{O}_{12},\mathcal{O}_{15}\}$
13	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu N$	$2\vec{S}_\chi\cdot v^\perp+2i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_N})$	$\{\mathcal{O}_8,\mathcal{O}_9\}$
14	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$	$\mathcal{O}_9$
15	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4\vec{S}_\chi\cdot\vec{S}_N$ $\rightarrow$ Standard SD	$\mathcal{O}_4$
16	$i\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i(\vec{S}_\chi\cdot\vec{v}^\perp)(\vec{S}_N\cdot\frac{\vec{q}}{m_M})$	$\mathcal{O}_{13}$
17	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu N$	$2i\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi$	$\mathcal{O}_{11}$
18	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi)[i\frac{\vec{q}^2}{m_N m_M}-4\vec{v}^\perp\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)]$	$\{\mathcal{O}_{11},\mathcal{O}_{15}\}$
19	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4i(\vec{S}_\chi\cdot\frac{\vec{q}}{m_M})(\vec{S}_N\cdot\vec{v}^\perp)$	$\mathcal{O}_{14}$
20	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi)(\frac{\vec{q}}{m_M}\cdot\vec{S}_N)$	$\mathcal{O}_6$

- There are 14 independent effective operators satisfying Galilean invariance, CPT symmetry, Hermitian
- Built out of the following four quantities

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{S}_{\chi}, \quad \vec{S}_N. \tag{4}$$

where $\vec{v}^{\perp} = \vec{v} + \vec{q}/2\mu_{\chi N}$, which satisfies $\vec{v}^{\perp} \cdot q = 0$.

WIMPs interact with bound nucleon in a nucleus

The nuclear dynamics, notably the relativistic mean-fields, may modify these effective operators

M. Bolsterli *et al.*, Phys. Rev. C **10** (1974) 1225.

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M. Bolsterli *et al.*, Phys. Rev. C **10** (1974) 1225.

Relativistic mean-field corrections

J. V. Noble, Phys. Rev. Lett. **43** (1979) 100.

In a nuclear medium, we assume that the wave function of a nucleon bound by Lorentz scalar and vector mean-fields satisfies the relativistic Dirac equation

$$\gamma^0\Big[-i\vec{\gamma}\cdot\nabla + M + V_s + \gamma^0 V_v\Big]\psi(\vec{x}) = E\psi(\vec{x})\,, \qquad (5)$$

where V_s corresponds to an attractive Lorentz scalar potential and V_v the repulsive fourth component of a four-vector. Although model-dependent, the typical values

$$V_s \approx -295\ \mathrm{MeV}, \qquad V_v \approx 222\ \mathrm{MeV}$$

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \begin{cases} u = \frac{1}{E-M-V_s-V_v}\vec{\sigma}\cdot\vec{p}v \\ v = \frac{1}{E+M+V_s-V_v}\vec{\sigma}\cdot\vec{p}u \to \frac{1}{2M[1+(V_s-V_v)/2M]}\vec{\sigma}\cdot\vec{p}u \end{cases}$$

j	$\mathcal{L}^j_{\rm int}$	Non-relativistic Reduction in medium ($u^\dagger \mathcal{O}_{eff} u$)
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N\frac{1}{1+(V_s-V_v)/2m_N}$
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-(\frac{\vec q}{m_\chi}\cdot\vec S_\chi)(\frac{\vec q}{m_N}\cdot\vec S_N)\frac{1}{1+(V_s-V_v)/2m_N}$
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q^2}{2m_N m_M}1_\chi 1_N+2(\frac{\vec q}{m_\chi}\times\vec S_\chi+i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$ $-[\frac{\vec q^2}{2m_N m_M}1_\chi 1_N-2i\frac{\vec k'+\vec k}{2m_N}\cdot(\frac{\vec q}{m_M}\times\vec S_N)]\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp+2i\vec S_\chi\cdot(\vec S_N\times\frac{\vec q}{m_\chi})-\vec S_N\cdot\frac{\vec k'+\vec k}{m_N}\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
8	$\bar\chi i\gamma^\mu\chi\bar N\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_M}\cdot\vec S_N$
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^2}{2m_\chi m_M}1_\chi 1_N-2(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_N}\times\vec S_N+i\vec v^\perp)$ $+2(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_N}\times\vec S_N-i\frac{\vec k'+\vec k}{2m_N})\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$

11	$\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}\gamma_\mu\gamma_5 N$	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_N$
12	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec{q}^2}{m_\chi m_M} - 4\vec{v}^\perp\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)](\frac{\vec{q}}{m_M}\cdot\vec{S}_N)$ $+4[\frac{\vec{k}'+\vec{k}}{2m_N}\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)(\frac{\vec{q}}{m_M}\cdot\vec{S}_N) - \frac{\vec{k}^2-\vec{k}'^2}{2m_N m_M}(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_N]\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
13	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu N$	$2\vec{S}_\chi\cdot v^\perp + 2i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_N}) + [\vec{S}_\chi\cdot\frac{\vec{k}'+\vec{k}}{m_N} - 2i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_N})]\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
14	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$
15	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4\vec{S}_\chi\cdot\vec{S}_N$
16	$i\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i(\vec{S}_\chi\cdot\vec{v}^\perp)(\vec{S}_N\cdot\frac{\vec{q}}{m_M}) + 4i[(\vec{S}_\chi\cdot\frac{\vec{k}'+\vec{k}}{2m_N})(\vec{S}_N\cdot\frac{\vec{q}}{m_M}) - \frac{\vec{k}'^2-\vec{k}^2}{2m_N m_M}(\vec{S}_\chi\cdot\vec{S}_N)]\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
17	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu N$	$2i\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi$
18	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi)[i\frac{\vec{q}^2}{m_N m_M} - 4\vec{v}^\perp\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)]$ $-(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi)[i\frac{\vec{q}^2}{m_N m_M} + 4\frac{\vec{k}'+\vec{k}}{2m_N}\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)]\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
19	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4i(\vec{S}_\chi\cdot\frac{\vec{q}}{m_M})(\vec{S}_N\cdot\vec{v}^\perp) - 4i(\vec{S}_\chi\cdot\frac{\vec{q}}{m_M})(\vec{S}_N\cdot\frac{\vec{k}'+\vec{k}}{2m_N})\frac{V_s-V_v}{2m_N[1+(V_s-V_v)/2m_N]}$
20	$i\bar{\chi} i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar{N} i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi)(\frac{\vec{q}}{m_M}\cdot\vec{S}_N)$

Summary

- The standard SI/SD operators are unchanged
- Certain interactions will receive non-negligible corrections ($25\% \sim 30\%$), which may significantly change the sensitivity of WIMP-nucleus cross sections to these operators
- It may help to study the nucleus-dependence of direct detection experiments
- This analysis can also be applied to the case of arbitrary dark matter spin, as well as inelastic scattering

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